HEAT EXCHANGE IN AN "INERTIAL" SOLAR COLLECTOR

A. V. Sinitsa and A. I. Tyutyunnikov

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Results are presented from a study of nonsteady heat exchange in a solar collector built into the structure of a building. Analytical expressions are obtained which permit modeling thermal processes in such devices.

Introduction. One of the promising approaches to providing competitive solar heat supply systems is the use of so-called "inertial" solar collectors [1]. These devices are made in a manner such that the heat collector is in direct contact with a substrate (foundation, roof, building panel, etc.) and an integral part thereof, combining the functions of a solar collector, heat storage device, and structural support.

Known methods for calculating solar collectors [2] based on the assumption of "zero" heat capacity of the collector construction are not always applicable in such cases. This is true because the nonsteady character of heat transport into the substrate mass may have a significant effect on heat exchange in inertial collectors.

Thus, heat exchange in an inertial collector must be considered with the substrate mass included in the object, which in the final reckoning leads to a complex problem of heat exchange of the liquid circulating in the collector channel with the external medium on the one hand, and with the substrate mass on the other.

In the general case the heat collector of the inertial collector may be of arbitrary form, which complicates or makes impossible analytical solution. For engineering calculations it is possible in principle to use the nonsteady analog of the solar collector efficiency coefficient [3] to reduce the problem of heat exchange in a collector of arbitrary form to the problem of heat exchange with a semi-infinite liquid mass moving in a fictitious slot channel.

Mathematical Formulation of the Problem. The object of study is a heat collector with a slot channel, located in ideal contact with a semi-infinite mass (Fig. 1). In formulating the mathematical model a number of assumptions are made, the most basic of which reduce to the following: the effect of the channel walls on heat exchange is neglected; transport and heat liberation coefficients are constant; as compared to the storage mass the thermo-

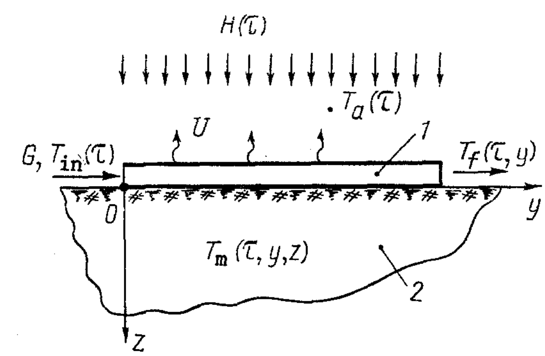

Fig. 1. Calculation scheme for inertial solar collector: 1) heat collector with slotted channel; 2) semi-infinite mass.

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TABLE 1. Forms of Transform Approximations

Type of transform	First-order approximation $K(s,\xi)\simeq\frac{a_0\sqrt{s}+b}{a_0\sqrt{s}+1}$ (*)	First-order approx. with delay term. $K(s,\xi)\simeq\frac{k}{\mathrm{Bi}+\sqrt{s}}\exp(-\lambda_0\sqrt{s})$ (**)
Equilibrium air temp.	$G_1^*(\mathrm{Fo},\xi)=1-\exp(-\xi)+A_1\mathcal{E}\left(\frac{\mathrm{Bi}\sqrt{\mathrm{Fo}}}{1+\varphi}\right)-A_2\mathcal{E}\left(\frac{\mathrm{Bi}\sqrt{\mathrm{Fo}}}{a_0}\right)$, $A_1=\frac{\varphi}{1+\varphi}\left\{1+\left(\exp[-\xi(1+\varphi)]\times\left[\exp(\xi\varphi)-\frac{a_0}{1+\varphi}\right]\right)\left(\frac{a_0}{1+\varphi}-1\right)^{-1}\right\}$; $A_2=\frac{\exp(-\xi)[1-\exp(-\xi\varphi)](a_0-1)}{(1+\varphi)\left(\frac{a_0}{1+\varphi}-1\right)}$	$G_1^{**}(\mathrm{Fo},\xi)=1-\frac{\varphi}{1+\varphi}\left\{\mathcal{E}\times\left(\frac{\mathrm{Bi}\sqrt{\mathrm{Fo}}}{1+\varphi}\right)-\exp\left(-\xi-\frac{\lambda_0^2}{4\mathrm{Fo}}\right)\times\left[\mathcal{E}\left(\frac{\lambda_0}{2\sqrt{\mathrm{Fo}}}\right)-\mathcal{E}\left(\frac{\lambda_0}{2\sqrt{\mathrm{Fo}}}+\frac{\mathrm{Bi}\sqrt{\mathrm{Fo}}}{1+\varphi}\right)\right]\right\}$, $\lambda_0=\frac{(\xi\varphi-1)}{\mathrm{Bi}}$
Heat exchange fluid input temp.	$G_2^*(\mathrm{Fo},\xi)=\exp(-\xi)\left\{1-[1-\exp(-\xi\varphi)]\,\mathcal{E}\left(\frac{\mathrm{Bi}\sqrt{\mathrm{Fo}}}{a_0}\right)\right\}$, $a_0=[1-\exp(-\xi\varphi)]^{-1}$; $\mathcal{E}(x)=\exp(x^2)\,\mathrm{erfc}(x)$; $\mathrm{erfc}(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty\exp(-t^2)\,dt$	$G_2^{**}(\mathrm{Fo},\xi)=\exp\left(-\xi-\frac{\lambda_0}{4\mathrm{Fo}}\right)\times\left[\mathcal{E}\left(\frac{\lambda_0}{2\sqrt{\mathrm{Fo}}}\right)-\mathcal{E}\left(\frac{\lambda_0}{2\sqrt{\mathrm{Fo}}}+\frac{\mathrm{Bi}\sqrt{\mathrm{Fo}}}{1+\varphi}\right)\right]$

inertial properties of the heat exchange fluid are negligible; heat transport within the mass in the direction of exchange fluid flow is negligibly low.

With consideration of these assumptions heat exchange in the inertial collector can be described by a system of differential equations in partial derivatives characterizing heat transport by the liquid and the mass, with matching conditions on the boundary.

For the mass:

$$\frac{\partial\Theta_m}{\partial\,\mathrm{Fo}}=\frac{\partial^2\Theta_m}{\partial\bar{z}^2};\ \Theta_m(0,\xi,\bar{z})=0;$$

$$\left.\frac{\partial\Theta_m}{\partial\bar{z}}\right|_{\bar{z}\to\infty}=0;\quad \left.\frac{\partial\Theta_m}{\partial\bar{z}}\right|_{\bar{z}=0}=\mathrm{Bi}\,(\Theta_m|_{\bar{z}=0}-\Theta_f). \tag{1}$$

For the liquid:

$$\frac{\partial\Theta_f}{\partial\xi}=\Theta_e-\Theta_f-\varphi\,(\Theta_m|_{\bar{z}=0}-\Theta_f);$$

$$\Theta_f(\mathrm{Fo},\ 0)=\Theta_{in}(\mathrm{Fo}). \tag{2}$$

Solution. By applying a Laplace transform [4] to system (1), (2) we reduce the problem to solution of the inhomogeneous differential equation

$$\frac{d\bar{\Theta}_f}{d\xi}=\bar{\Theta}_e-\bar{\Theta}_f\left(1+\frac{\varphi\sqrt{s}}{\mathrm{Bi}+\sqrt{s}}\right) \tag{3}$$

with

$$\bar{\Theta}_f(s,\ 0)=\bar{\Theta}_{in}(s),$$

where $\bar{\Theta}_f$, $\bar{\Theta}_e$, $\bar{\Theta}_{in}$ are Laplace transforms of the functions describing change in the temperature of the heat exchange fluid, the equilibrium temperature, and the temperature at the inertial collector input.

TABLE 2. Estimate of Integral Error of Approximation of Transcendental form K(s, ξ) (%)

Form of K(s, ξ) approximation	ξφ			
	1	2	5	10
$W_1 = \frac{a_0\sqrt{s}+b}{a_0\sqrt{s}+1}$ (*)	−5,46	−15,65	−43,82	−66,67
$W_2 = \frac{k\exp(-\lambda_0\sqrt{s})}{\mathrm{Bi}+\sqrt{s}}$ (**)	+33,3	+37,5	+25,71	+15,83

Form of K(s, ξ) approximation	ξφ			
	20	40	60	100
$W_1 = \frac{a_0\sqrt{s}+b}{a_0\sqrt{s}+1}$ (*)	−81,82	−90,48	−93,55	−96,08
$W_2 = \frac{k\exp(-\lambda_0\sqrt{s})}{\mathrm{Bi}+\sqrt{s}}$ (**)	+8,86	+4,70	+3,20	+1,95

The solution of Eq. (3) has the form, known from [5]

$$\bar{\Theta}_f(s, \xi) = s\bar{\Theta}_e(s)\bar{G}_1(s, \xi) + s\bar{\Theta}_{in}(s)\bar{G}_2(s, \xi), \tag{4}$$

where

$$\bar{G}_1(s, \xi) = \frac{1 - \exp\left[-\xi\left(1 + \frac{\varphi\sqrt{s}}{\mathrm{Bi}+\sqrt{s}}\right)\right]}{s\left(1 + \frac{\varphi\sqrt{s}}{\mathrm{Bi}+\sqrt{s}}\right)}$$

is the transform for the disturbance in equilibrium temperature of the external air;

$$\bar{G}_2(s, \xi) = \frac{1}{s}\exp\left[-\xi\left(1 + \frac{\varphi\sqrt{s}}{\mathrm{Bi}+\sqrt{s}}\right)\right]$$

is the transform for the heat exchange fluid input temperature.

To find the originals of the transforms we use an operational method based on the Efros theorem [4] which provides a generalized relationship for calculating the originals.

The transform for unit change in equilibrium temperature of the external air is

$$G_1(\mathrm{Fo}, \xi) = 1 - \exp(-\xi) - \frac{\exp[-\xi(1+\varphi)]}{(1+\varphi)}\sum_{n=1}^{\infty}\left(\frac{\varphi}{1+\varphi}\right)^n \times$$
$$\times \sum_{k=1}^{n}\frac{[\xi(1+\varphi)]^k}{k!}\sum_{m=0}^{n-1}(2\,\mathrm{Bi}\sqrt{\mathrm{Fo}})^m \exp(\mathrm{Bi}^2\mathrm{Fo})\,\mathrm{i}^m\,\mathrm{erfc}(\mathrm{Bi}\sqrt{\mathrm{Fo}}), \tag{5}$$

where

$$\mathrm{i}^m\,\mathrm{erfc}(x) = \int_x^{\infty}\mathrm{i}^{m-1}\,\mathrm{erfc}(y)\,dy$$

is the integral error function [6].

The transform for unit change in fluid input temperature is

$$G_2(\mathrm{Fo}, \xi) = \exp(-\xi) - \exp[-\xi(1+\varphi)]\sum_{n=1}^{\infty}\frac{(\xi\varphi)^n}{n!}\sum_{m=0}^{n-1}(2\,\mathrm{Bi}\sqrt{\mathrm{Fo}})^m\exp(\mathrm{Bi}^2\,\mathrm{Fo})\,\mathrm{i}^m\,\mathrm{erfc}(\mathrm{Bi}\sqrt{\mathrm{Fo}}). \tag{6}$$

It is of interest to solve the problem of heat exchange in the inertial collector for a known initial temperature distribution in the substrate mass, where $\Theta_m(0, \bar{z}) = \Phi_0(\bar{z})$. In this case the expressions for the transforms (5), (6) remain unchanged, while the inhomogeneous initial temperature distribution in the mass appears in the form of an additional heat source, uniform along the channel length, with temperature

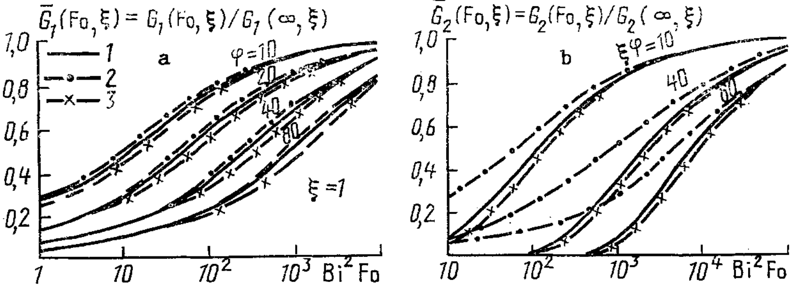

Fig. 2. Comparison of transforms for various approximation forms: a) for unit change in equilibrium air temperature; b) for unit change in heat exchange fluid input temperature: 1, exact value; 2, approximate transform for K(s, ξ) approximation in form (*); 3, approximate transform for K(s, ξ) approximation in form (**).

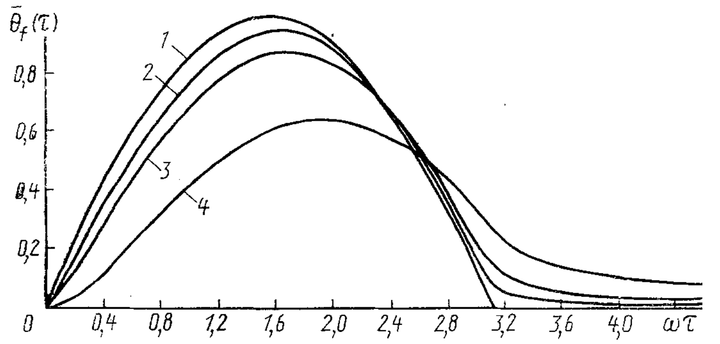

Fig. 3. Change in relative temperature at output of "inertial" solar collector $\bar{\Theta}_f(\tau) = \Theta_f(\tau)/(A \cdot E_{sc})$ for coating heat transfer coefficient ε = 8 W/(m²·K) and thermal activity coefficient ε = 2000 W·sec$^{1/2}$/(m²·K): 1, $\bar{\Theta}_f^{stat}(\tau) = \sin\omega\tau + \sin(\omega\tau - \pi)\,\sigma(\omega\tau - \pi)$ j 2, $\bar{\Theta}_f$ for gc_p = 100 W/(m²·K); 3, $\bar{\Theta}_f$ for gc_p = 40 W/(m²·K); 4, $\bar{\Theta}_f$ for gc_p = 10 W/(m² × K).

$$\Theta_{s}(\mathrm{Fo}) = \int_0^\infty \Phi_0(\bar{z})\, V(\mathrm{Fo},\ \bar{z})\, d\bar{z}, \qquad (7)$$

where

$$V(\mathrm{Fo},\ \bar{z}) = \exp\left[-\left(\frac{\bar{z}}{2\sqrt{\mathrm{Fo}}}\right)^2\right]\left[\frac{1}{\sqrt{\pi\,\mathrm{Fo}}} - \mathrm{Bi}\exp(\mathrm{Bi}^2\,\mathrm{Fo})\,\mathrm{erfc}(\mathrm{Bi}\sqrt{\mathrm{Fo}})\right]$$

is the source function on the surface of the semi-infinite mass [4].

With consideration of Eqs. (5)-(7), on the basis of Duhamel's principle [4] the general solution of the heat exchange problem in an inertial collector with arbitrary perturbing effects and inhomogeneous temperature field in the substrate has the form

$$\Theta_f(\mathrm{Fo},\ \xi) = \frac{\partial}{\partial\,\mathrm{Fo}}\left\{\int_0^{\mathrm{Fo}} [\Theta_e(t) + \Theta\ \ (t)]\, G_1(\mathrm{Fo} - t,\ \xi)\, dt + \int_0^{\mathrm{Fo}} \Theta_{in}(t)\, G_2(\mathrm{Fo} - t,\ \xi)\, dt\right\}. \qquad (8)$$

Use of the equations obtained for modeling inertial collector operation is inconvenient because of their cumbersomeness and the complexity of the calculation procedures, especially for long time intervals.

Approximation. The complex form of the transforms (5), (6) is related to the fact that within the space of the transforms the functions in Eq. (4) contain the transcendental form $K(s,\ \xi) = \exp[\xi\varphi\mathrm{Bi}/(\mathrm{Bi} + \sqrt{s})]$, which has no original in closed form.

On the basis of the theory of asymptotically equivalent functions [7] and the integral evaluation method of [8] various approximations of K(s, ξ) were considered (Table 1). The free coefficients in the approximation expressions were determined by the condition of equality of the exact and approximate expressions as $s \to 0$ and $s \to \infty$, as well as equality of the derivatives $s \to 0$ [8]. The integral evaluation method permits determination of the integral error of one or the other approximation without transition to the original from the condition

$$\delta = 1 - \left.\frac{W''(p, \xi)}{K''(p, \xi)}\right|_{p=0},$$

where W(p, ξ) is an expression approximating the transcendent form K(p, ξ); $p = \sqrt{s}$.

The character of the change in integral error for various approximate forms as a function of the regime parameter $\xi\varphi$ can be judged from the results presented in Table 2.

In the majority of practical cases $\xi\varphi \geq 10$, and for such values of the regime parameter the best approximation of the exact solution is provided by the first order approximation of the form K(s, ξ) with a delay term, producing not only the lowest values of the integral, but also the dynamic error (Fig. 2).

Studies of transient processes in the inertial collector by the laboratory model showed satisfactory convergence of theoretical and experimental results.

Modeling. For the majority of practical problems in calculating solar apparatus the heat exchange agent temperature at the input to the collector can be considered close to zero ($\Theta_{in}(\tau) = 0$), while the equilibrium temperature of the outside air is described well by a semi-sinusoidal pulse with semiperiod close to the solstice time:

$$\Theta_e(\tau) = A \sin(\omega\tau)\,[1 - \sigma(\omega\tau - \pi)],$$

where A, ω are the amplitude and frequency of the oscillation; $\sigma(\omega\tau - \pi)$ is a unit Heaviside function.

Modeling results for this case with a comparison of temperature values at the outputs of an "inertial" and inertialess (with "zero" heat capacity) solar collectors are shown in Fig. 3. For the latter collector the temperature at the output is described by the expression known from [2]

$$\Theta_f^{stat}(\tau) = \Theta_e(\tau)\left[1 - \exp\left(-\frac{F'U}{gc_e}\right)\right].$$

Analysis of the change in relative temperature values over time of Fig. 3 shows that with increase in specific water equivalent gc_p the output temperature of the "inertial" collector approaches the corresponding value for the inertialess collector. On the other hand, with decrease in gc_p the amplitude of oscillations in output temperature of the inertial collector decreases, but accumulation of heat in the collector substrate insures heating of the transfer fluid in the absence of solar radiation in the period $\omega\tau > \pi$. For a very low fluid flow rate a state sets in which practically total extinction in oscillations of the fluid temperature is found.

The limiting states of inertial solar collector operation can be clearly seen from a frequency diagram (Fig. 4), in which the collector is interpreted as a low pass filter. At oscillation frequencies $\Theta_p(\tau)$ higher than the cutoff frequency the amplitude of temperature oscillations at the collector output is close to zero (region I). Region III is characterized by a quasi-steady operating regime in which the collector can be considered as having "zero" heat capacity. The limiting frequencies and operating regimes of the collector are then determined to an equal degree by its thermotechnical properties (ε and F'U) and the regime parameter gc_p.

Conclusion. The studies of heat exchange in an "inertial" collector performed above have obtained exact and approximate expressions for transforms which permit modeling of operation for the purpose of optimizing construction and regime parameters. The solutions of Eqs. (5), (6) are applicable in the case of a limited substrate given the condition that its thickness $b > 2\sqrt{3a\tau}$ [9].

NOTATION

$T_m = T_m(\tau, y, z)$, temperature of mass; T_f, $T_f(\tau, y)$, temperature of heat exchange fluid; H(τ), density of solar radiation absorbed by heat collector; U, thermal loss coefficient of solar

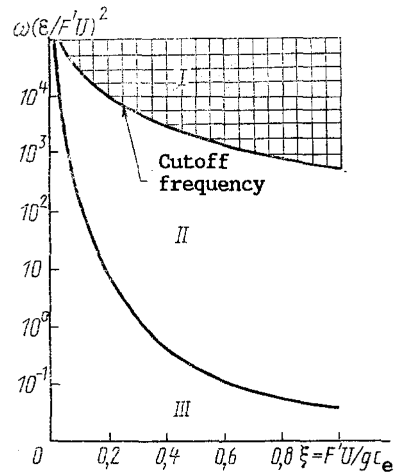

Fig. 4. Frequency diagram of "inertial" solar collector: 1) cutoff range; 2) nonsteady heat exchange range; 3) quasi-steady heat exchange range.

collector; $T_a(\tau)$, external air temperature; $T_e = T_e(\tau) = H(\tau)/U + T_a(\tau)$, equilibrium temperature of external air; y, z, spatial coordinates; τ, time; a, λ, thermal diffusivity and conductivity of mass; α, heat exchange coefficient between heat exchange fluid and channel; $F' = \alpha/(\alpha + U)$, efficiency of solar collector with slotted channel; G, mass flow rate of heat exchange fluid; c_e, heat capacity of heat exchange fluid; W, ℓ, width and length of heat collector; $gc_e = Gc_e/W\ell$, specific water equivalent of heat exchange fluid; $\bar{y} = y/\ell$, $\bar{z} = z/\ell$, relative spatial coordinates; $\xi = \bar{y}\cdot F'U/gc_e$, generalized longitudinal coordinate; $Bi = \alpha\ell/\lambda$, Biot number; $Fo = a\tau/\ell^2$, Fourier number; s, Laplace variable; $\Theta_m = T_m - T_0$; $\Theta_{in} = T_{in} - T_0$; $\Theta_f = T_f - T_0$; $\Theta_e = T_e - T_0$; T_0, initial temperature of mass; b, plate thickness; $E_{sc} = 1 - \exp(-F'U/gc_e)$, solar collector efficiency; ε, substrate thermal activity coefficient.

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